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# Constraining an $R$ -parity violating supersymmetric theory from the SuperKamiokande data on atmospheric neutrinos

Aseshkrishna Datta <sup>1</sup>, Biswarup Mukhopadhyaya<sup>2</sup> and Sourov Roy <sup>3</sup>

*Mehta Research Institute,*

*Chhatnag Road, Jhusi, Allahabad - 211 019, India,*

## Abstract

The constraints on an  $R$ -parity violating supersymmetric theory arising from the recent SuperKamiokande results on atmospheric neutrinos are studied, with special reference to a scenario with bilinear  $R$ -parity violating terms. Considering both the fermionic and scalar sectors, we find that a large area of the parameter space is allowed, in terms of both the lepton-number violating entries in the superpotential and the soft  $R$ -violating terms in the scalar potential, and that no fine-tuning is required. However, the need to avoid flavour changing neutral currents puts additional restrictions on the theory, requiring either the  $R$ -violating terms in the superpotential to be smaller than the  $R$ -conserving ones, or a hierarchy in the  $R$ -violating parameters for different lepton flavours in the superpotential.

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<sup>1</sup>E-mail: asesh@mri.ernet.in

<sup>2</sup>E-mail: biswarup@mri.ernet.in

<sup>3</sup>E-mail: sourov@mri.ernet.in

# 1 Introduction

Ever since the evidence in favour of neutrino oscillation has received reinforcement from the SuperKamiokande(SK) results on the atmospheric  $\nu_\mu$ -deficiency [1], intensive discussions have taken place on ‘new physics’ that can give rise to neutrino masses and mixing of the expected types. In the simplest explanation, the deficiency is due to the oscillation of  $\nu_\mu$  into  $\nu_\tau$ , with  $\Delta m^2 \simeq 10^{-3}$  eV<sup>2</sup> and large angle vacuum mixing ( $\sin^2 2\theta > 0.8$ ) between the two neutrino species. Such oscillation is also consistent with the results from the SOUDAN [2] and MACRO [3] experiments. Side by side, one also faces the requirement of accounting for the solar neutrino deficit [4]. Using the still available explanations in terms of  $\nu_e - \nu_\mu$  oscillation, this means a mass-squared difference of about  $10^{-5} - 10^{-6}$  eV<sup>2</sup> for the MSW solution (with either small or large angle mixing), or a mass-squared splitting smaller by about 4-5 orders for the vacuum oscillation solution. Thus one faces the task of reconciling a mass hierarchy with large angle mixing between the two heaviest neutrino species. In whichever way that is possible, one has to step outside the domain of the standard electroweak model.

Supersymmetry (SUSY) has been studied for a long time now, from both theoretical and phenomenological angles, as one of the most attractive options beyond the standard model [5]. And yet, apart from indirect successes such as offering solutions to the hierarchy puzzle, it has not been possible to confront SUSY with any clear experimental results so far. It is therefore appropriate that when the atmospheric neutrino results are so emphatically underlining the existence of neutrino masses and mixing with a given pattern, the relevance of SUSY in generating and explaining such a pattern should be thoroughly explored.

If the SUSY extension of the standard model (SM) with the minimal particle content has to be invoked for the purpose, then the absence of any right-handed neutrino (and therefore of Dirac masses) in the scenario immediately points towards Majorana neutrinos as the likely solution. However, the latter implies the violation of lepton number by 2 units. That again can come rather naturally in a scenario where R-parity, defined as  $R = (-1)^{3\mathcal{B} + \mathcal{L} + 2\mathcal{S}}$ , is violated [6]. In such a scenario, the  $\Delta\mathcal{L} = 1$  terms in the Lagrangian can ultimately give rise to Majorana masses either through a tree-level see-saw type mechanism or via loop effects. The ‘attractive’ point here is that one does not have to postulate the existence of any particle *specifically* for the generation of neutrino masses, since the superparticles that are indispensable components of a SUSY theory are sufficient for the purpose.

Perhaps the most convenient and cogent (though not unique) way of introducing R-parity violation is an extension of the superpotential, using trinilinear and/or bilinear terms. Trinilinear terms in the superpotential [7] give rise to  $\mathcal{L}$  (or  $\mathcal{B}$ )-violating Yukawa-type interactions and trilinear soft terms in the scalar potential. The presence of bilinear  $\mathcal{L}$ -violating terms [8, 9, 10], on the other hand, are generally responsible for non-vanishing vacuum expectation values (vev) for sneutrinos, which also lead to mixing between neutrinos and neutralinos as also between charged leptons and charginos (and similarly between the Higgs and charged slepton/sneutrino states in the scalar

sector). The bearing of both approaches on neutrino masses have been studied extensively in recent times, whereby constraints on the R-parity violating parameters from neutrino masses have been derived [11, 12]. One also finds in the literature discussions on how to test the consequences of the corresponding scenarios in accelerator-based experiments [13].

In several of these references, it was shown how one could accommodate the mass hierarchy together with large angle mixing between  $\nu_\mu$  and  $\nu_\tau$  through neutrino-neutralino mixing via the bilinear terms mentioned above, while the smaller mass splitting between the  $\nu_\mu$  and  $\nu_e$  could be due to loop-induced effects. However, one can still ask a number of questions related to the parameter space of the theory, extending both over the fermionic and the scalar sector, before the SUSY explanation of the SK results can acquire sufficient credibility. In this paper we have tried to find answers to some such questions, and to establish that the solution space for the SK deficits is *not* a fine-tuned one.

To be more specific, our analysis includes both the scalar and spin-1/2 sectors of an R-parity violating scenario (where the simplest, bilinear R-violating terms are introduced as necessary ingredients but no generality is otherwise discarded) which can explain the SK data. Keeping the value of  $\Delta m_{\mu\tau}^2$  and the large mixing angle as inputs, we have tried to find as general answers as possible to the following questions:

- Is it a necessity to have a large hierarchy between the bilinear terms corresponding to different lepton families in the superpotential?
- Is it required to have a hierarchy between soft  $\mathcal{L}$ -violating terms involving different lepton flavours?
- How crucial is it to have a hierarchy of orders between the  $\mathcal{L}$ -conserving and  $\mathcal{L}$ -violating bilinear terms in the superpotential and the scalar potential?
- How is the suppression of flavour changing neutral currents (FCNC) ensured in such a picture?

In this analysis, we have used low-energy values of all the parameters in the theory, and have not attempted to link them to any specific high-scale physics. However, our chosen convention of writing the soft bilinear terms in the scalar potential carries some influence of a supergravity (SUGRA)-based model where such terms can be shown to originate from an interference of terms belonging to the hidden and observable sectors in the superpotential.

Section 2 sets up the general framework within which we operate, incorporating the fermion and scalar mixing schemes. The different choices of basis in which we have worked are specified there, and the parameters which we ultimately use to explore large angle mixing in the most general sense are defined. Section 3 contains numerical studies of the constraints on the theory in terms of these parameters. This gives us full or partial answers to the questions listed above, obtained either from detailed calculations or from simple estimates of order. We conclude in section 4.

## 2 The Framework

The MSSM superpotential is given by (suppressing the  $SU(2)$  indices)

$$W_{MSSM} = \mu \hat{H}_1 \hat{H}_2 + h_{ij}^l \hat{L}_i \hat{H}_1 \hat{E}_j^c + h_{ij}^d \hat{Q}_i \hat{H}_1 \hat{D}_j^c + h_{ij}^u \hat{Q}_i \hat{H}_2 \hat{U}_j^c \quad (1)$$

where  $\mu$  is the Higgsino mass parameter and the last three terms give all the Yukawa interactions.

When R-parity is violated, the following additional terms can be added to the superpotential:

$$W_R = \lambda_{ijk} \hat{L}_i \hat{L}_j \hat{E}_k^c + \lambda'_{ijk} \hat{L}_i \hat{Q}_j \hat{D}_k^c + \lambda''_{ijk} \hat{U}_i^c \hat{D}_j^c \hat{D}_k^c + \epsilon_i \hat{L}_i \hat{H}_2 \quad (2)$$

Where the  $\lambda''$ -terms correspond to  $\mathcal{B}$ -violation, and the remaining ones, to  $\mathcal{L}$ -violation. The absence of proton decay makes it customary to have one of these two types of nonconservation at a time. In the rest of this discussion, we shall not consider  $\mathcal{B}$ -violation.

The  $\lambda$ -and  $\lambda'$ -type terms have been widely studied; their contributions to neutrino masses can be only through loops, and their multitude (there are 36 of them altogether) makes the necessary adjustments possible for creating the required values of neutrino masses and mixing angles.

More interesting, however, are the three bilinear terms  $\epsilon_i L_i H_2$ . It is in fact quite useful to use them as the starting inputs of R-parity violation in the theory. There being at most only three parameters of this type, the model looks much more predictive than one with 36 unrelated trilinear terms. Furthermore, the physical effects of the trilinear terms can be generated from the bilinears, by going to the appropriate bases. There are additional interesting consequences of the bilinear terms. The presence of the  $LH_2$ -term means a mixing between the Higgsinos and charged and neutral lepton states. In addition, the scalar potential in such a case contains terms bilinear in the slepton and the higgs fields and involving only second and third generation of sleptons (the reason behind this assumption will be clarified as we proceed) the terms are as follows :

$$\begin{aligned} V_{scal} = & m_{L_3}^2 \tilde{L}_3^2 + m_{L_2}^2 \tilde{L}_2^2 + m_1^2 H_1^2 + m_2^2 H_2^2 + B\mu H_1 H_2 \\ & + B_2 \epsilon_2 \tilde{L}_2 H_2 + B_3 \epsilon_3 \tilde{L}_3 H_2 + \mu \epsilon_3 \tilde{L}_3 H_1 + \mu \epsilon_2 \tilde{L}_2 H_1 + \dots \end{aligned} \quad (3)$$

where  $m_{L_i}$  denotes the mass of the  $i$ -th scalar doublet,  $\tilde{L}_i = \begin{pmatrix} \tilde{\nu}_i \\ \tilde{l}_i \end{pmatrix}_L$ ,  $i$  being 2 and 3 for the second and third generations respectively, at the electroweak scale. Here  $m_{L_2}$  and  $m_{L_3}$  are the slepton mass parameters. In our subsequent analysis, all the sleptons (of both chiralities) and sneutrinos have been assumed to be degenerate.

An immediate consequence of the additional ( $\mathcal{L}$ -violating) soft terms in the scalar potential is a set of non-vanishing sneutrino vev's [14]. This is a characteristic feature of this scenario, which in addition produces neutrino(charged lepton)-gaugino mixing via the sneutrino-neutrino(charged lepton)-gaugino interaction terms. The former type of mixing leads to a neutrino mass at the tree-level.

Since our primary goal is to explain large angle  $\nu_\mu - \nu_\tau$  mixing, we simplify the picture by assuming that only the second and third generations enter into this tree-level mixing process. For this, we postulate only two bilinear R-violating terms, proportional to  $\epsilon_2$  and  $\epsilon_3$ . These two terms, together with the soft bilinear terms  $B_2$  and  $B_3$ , form the set of independent R-parity violating parameters in this basis, henceforth to be called *basis 1*. The vev's corresponding to the muonic and tau sneutrinos are  $v_\mu$  and  $v_\tau$  respectively in this basis. For reasons that will become apparent later, we choose to treat these vev's as independent parameters, and use them to derive values of the soft terms. For that purpose, one has to make use of the set of tadpole equations arising from electroweak symmetry breaking:

$$(m_1^2 + 2\lambda c)v_1 + B\mu v_2 + \mu\epsilon_2 v_\mu + \mu\epsilon_3 v_\tau = 0 \quad (4)$$

$$(m_2^2 - 2\lambda c)v_2 + B\mu v_1 + B_2\epsilon_2 v_\mu + B_3\epsilon_3 v_\tau = 0 \quad (5)$$

$$(m_{\tilde{\nu}_\mu}^2 + 2\lambda c)v_\mu + B_2\epsilon_2 v_2 + \mu\epsilon_2 v_1 + \epsilon_2\epsilon_3 v_\tau = 0 \quad (6)$$

$$(m_{\tilde{\nu}_\tau}^2 + 2\lambda c)v_\tau + B_3\epsilon_3 v_2 + \mu\epsilon_3 v_1 + \epsilon_2\epsilon_3 v_\mu = 0 \quad (7)$$

where  $v_1 = \langle H_1 \rangle$ ,  $v_2 = \langle H_2 \rangle$ ,  $c = (v_1^2 - v_2^2 + v_\tau^2 + v_\mu^2)$  and  $\lambda = (g^2 + g'^2)/8$ .

While two of these equations can be used to eliminate the soft Higgs mass terms  $m_1$  and  $m_2$ , the  $\mathcal{L}$ -violating soft terms  $B_2$  and  $B_3$  can be obtained from the remaining two:

$$B_2 = -\frac{1}{\epsilon_2 v_2}(\epsilon_2\epsilon_3 v_\tau + 2\lambda c v_\mu + m_{\tilde{\nu}_\mu}^2 v_\mu + \epsilon_2\mu v_1) \quad (8)$$

$$B_3 = -\frac{1}{\epsilon_3 v_2}(\epsilon_2\epsilon_3 v_\mu + 2\lambda c v_\tau + m_{\tilde{\nu}_\tau}^2 v_\tau + \epsilon_3\mu v_1) \quad (9)$$

where, again, the sneutrino masses have been assumed to be degenerate with a common slepton mass parameter.

The next step is to rotate away both the  $\epsilon$ -terms from the superpotential. In the process we go from the basis  $(H_1, L_3, L_2)$  to  $(H'_1, L'_3, L'_2)$  using the following rotation :

$$\begin{pmatrix} H'_1 \\ L'_3 \\ L'_2 \end{pmatrix} = \begin{pmatrix} c_3 & s_3 c_2 & s_3 s_2 \\ -s_3 & c_3 c_2 & c_3 s_2 \\ 0 & -s_2 & c_2 \end{pmatrix} \begin{pmatrix} H_1 \\ L_3 \\ L_2 \end{pmatrix} \quad (10)$$

where  $s_2 = \frac{\epsilon_2}{\sqrt{\epsilon_2^2 + \epsilon_3^2}}$ ,  $c_2 = \frac{\epsilon_3}{\sqrt{\epsilon_2^2 + \epsilon_3^2}}$ ,  $c_3 = \frac{\mu}{\mu'}$ ,  $s_3 = \frac{\sqrt{\epsilon_2^2 + \epsilon_3^2}}{\mu'}$ , and  $\mu' = \sqrt{\mu^2 + \epsilon_2^2 + \epsilon_3^2}$ . Clearly, this leaves  $\mu' H'_1 H_2$  as the only bilinear term in the superpotential. The physical consequences of bilinears R-parity violation, however, are still existent in this basis, since the scalar potential contains its signature, and one has now 'rotated' sneutrino vev's  $v'_\mu$  and  $v'_\tau$  which, together with the Higgs vev  $v'_1$ , are connected to the set  $(v_1, v_\tau, v_\mu)$  by the rotation matrix given in Eqn. (10) above.

In this basis (called *basis 2*), the neutralino mass matrix (which is of dimension  $6 \times 6$  after including the neutrinos) is given by

$$\mathcal{M}_\chi = \begin{pmatrix} 0 & -\mu' & \frac{gv_2}{\sqrt{2}} & -\frac{g'v_2}{\sqrt{2}} & 0 & 0 \\ -\mu' & 0 & -\frac{gv'_1}{\sqrt{2}} & \frac{g'v'_1}{\sqrt{2}} & 0 & 0 \\ \frac{gv_2}{\sqrt{2}} & -\frac{gv'_1}{\sqrt{2}} & M & 0 & -\frac{gv'_\tau}{\sqrt{2}} & -\frac{gv'_\mu}{\sqrt{2}} \\ -\frac{g'v_2}{\sqrt{2}} & \frac{g'v'_1}{\sqrt{2}} & 0 & M' & \frac{g'v'_\tau}{\sqrt{2}} & \frac{g'v'_\mu}{\sqrt{2}} \\ 0 & 0 & -\frac{gv'_\tau}{\sqrt{2}} & \frac{g'v'_\tau}{\sqrt{2}} & 0 & 0 \\ 0 & 0 & -\frac{gv'_\mu}{\sqrt{2}} & \frac{g'v'_\mu}{\sqrt{2}} & 0 & 0 \end{pmatrix} \quad (11)$$

where the successive rows and columns correspond to  $(\tilde{H}_2, \tilde{H}'_1, -i\tilde{W}_3, -i\tilde{B}, \nu'_\tau, \nu'_\mu)$ . Here  $M$  and  $M'$  are the SU(2) and U(1) gaugino mass parameters respectively,  $\mu'$  is the Higgsino mass parameter in basis 2 and  $v'_1 = \langle H'_1 \rangle$ . This leads to *one* non-vanishing tree-level neutrino mass eigenvalue. The root of this can be traced to the fact that one linear combination of  $\nu'_\mu$  and  $\nu'_\tau$ , given by

$$\nu_3 = \nu'_\tau \cos \theta + \nu'_\mu \sin \theta \quad (12)$$

enters into cross-terms with the  $\tilde{B}$  and  $\tilde{W}_3$ , while its orthogonal state  $\nu_2$  decouples from the mass matrix and therefore remains massless [15]. The angle  $\theta$  is given by

$$\cos \theta = v'_\tau / v' \quad (13)$$

with  $v' = \sqrt{v'^2_\mu + v'^2_\tau}$ . The quantity  $v'$ , which is a kind of ‘effective’ sneutrino vev in a basis where the bilinear terms are rotated away from the superpotential, is a basis-independent measure of R-parity violation, which directly controls the tree-level neutrino mass acquired in the process. The mass implied by the atmospheric  $\nu_\mu$ -deficit in the SK data requires  $v'$  to be in the range  $(1 - 3) \times 10^{-4}$  GeV approximately [16], depending on the mass of the lightest neutralino ( $\chi^0_1$ ). It is also interesting to note at this stage that the smallness of the neutrino mass does not limit the value of the  $\epsilon$ -parameters so long as  $v'$  lies within about 100 keV or so. It has been demonstrated, for example, that in theories based on  $N=1$  supergravity (SUGRA), it is indeed possible to have a small  $v'$  in spite of large values of the  $\epsilon$ ’s, by setting all bilinear soft terms (both  $\mathcal{L}$ -violating and conserving) to the same value at the SUGRA breaking scale [17].

Modulo the very small neutrino-neutralino mixing, the angle  $\theta$  defined above can be identified with the neutrino vacuum mixing angle ( $\theta_0$ ) if the *charged lepton mass matrix is diagonal* in basis 2. Only in such a case can we say that a near-equality of the vev’s  $v'_\mu$  and  $v'_\tau$  is required by the condition of large-angle mixing. However, the general form of the charged lepton mass matrix in this basis is

$$\mathcal{M}_l = \begin{pmatrix} f_3(-v'_\mu s_2 s_3 + c_2 v'_1) & f_2(s_2 v'_1 + c_2 s_3 v'_\mu) \\ f_3(-c_3 s_2 v'_1 + s_2 s_3 v'_\tau) & f_2(c_2 c_3 v'_1 - c_2 s_3 v'_\tau) \end{pmatrix} \quad (14)$$

Here we have assigned  $(\tau'_L, \mu'_L)$  along the rows and  $(\tau_R, \mu_R)$  along the columns. One should note that  $f_3 = h_{33}^l = \frac{m_\tau}{v_1}$  and  $f_2 = h_{22}^l = \frac{m_\mu}{v_1}$ . Thus  $\mathcal{M}_l$  has non-vanishing off-diagonal terms in general.

The conditions under which it can be approximately treated as diagonal (and the equality of the two sneutrino vev's in basis  $\mathcal{2}$  is a necessity) will be specified in the next section. Instead, we observe here that the actual neutrino mixing matrix is given by

$$V_l = V_\theta U^T \quad (15)$$

where  $V_\theta$  is the  $2 \times 2$  orthogonal matrix corresponding to the rotation angle  $\theta$ , and  $U$  is the matrix that diagonalises  $\mathcal{M}_l$  in basis  $\mathcal{2}$ . It is the matrix  $V_l$  which should correspond to a rotation angle of  $\pi/4$  for maximal mixing, and to a range approximately between 32 and 58 degrees for  $\sin^2 2\theta_0 > 0.8$ ,  $\theta_0$  being the neutrino mixing angle determined by  $V_l$ <sup>1</sup>. This restricts one to specific allowed ranges in the ratio of the vev's  $v'_\mu$  and  $v'_\tau$ , for each value of the ratio  $\epsilon_2/\epsilon_3$ . Corresponding to these allowed values, the 2-dimensional space in the soft parameters  $B_2$  and  $B_3$  also gets constrained. In the next section, we shall present a detailed map of the region of the parameter space which corresponds to large angle mixing as depicted by the SK data.

Before we end this section, we also list the three scalar mass matrices for this theory in basis  $\mathcal{1}$ . These include both the Higgs and the slepton/sneutrino sectors, with the appropriate kinds of mixing between them.

The charged scalar mass matrix is given by

$$M_c^2 = \begin{pmatrix} s + \alpha'_c + f_{3\tau}^2 & -B\mu + \alpha_{12} & \mu\epsilon_3 + \alpha_{1\tau} - \frac{f_{31}f_{3\tau}}{2} & \mu\epsilon_2 + \alpha_{1\mu} - \frac{f_{21}f_{2\mu}}{2} & -\epsilon_3f_{32} - Af_{3\tau} & -\epsilon_2f_{22} - Af_{2\mu} \\ -B\mu + \alpha_{12} & r - \alpha'_c & -B_3\epsilon_3 + \alpha_{2\tau} & -B_2\epsilon_2 + \alpha_{2\mu} & -\epsilon_3f_{31} & -\epsilon_2f_{21} \\ \mu\epsilon_3 + \alpha_{1\tau} - \frac{f_{31}f_{3\tau}}{2} & -B_3\epsilon_3 + \alpha_{2\tau} & p_\tau + \alpha_{t\tau} + \alpha'_c & \epsilon_2\epsilon_3 & \mu f_{32} + Af_{31} & 0 \\ \mu\epsilon_2 + \alpha_{1\mu} - \frac{f_{21}f_{2\mu}}{2} & -B_2\epsilon_2 + \alpha_{2\mu} & \epsilon_2\epsilon_3 & p_\mu + \alpha_{t\mu} + \alpha'_c & 0 & \mu f_{22} + Af_{21} \\ -\epsilon_3f_{32} - Af_{3\tau} & -\epsilon_3f_{31} & \mu f_{32} + Af_{31} & 0 & q_\tau - 2\alpha'_c + f_{3\tau}^2 & f_{2\mu}f_{3\tau} \\ -\epsilon_2f_{22} - Af_{2\mu} & -\epsilon_2f_{21} & 0 & \mu f_{22} + Af_{21} & f_{2\mu}f_{3\tau} & q_\mu - 2\alpha'_c + f_{2\mu}^2 \end{pmatrix} \quad (16)$$

with

$$r = m_2^2 + \frac{1}{4}g^2(v_1^2 + v_2^2 + v_\tau^2 + v_\mu^2)$$

$$s = m_1^2 + \frac{1}{4}g^2(v_1^2 + v_2^2 - v_\tau^2 - v_\mu^2)$$

$$p_\tau = m_{\tilde{\tau}_L}^2 + f_{31}^2$$

$$q_\tau = m_{\tilde{\tau}_R}^2 + f_{31}^2$$

$$p_\mu = m_{\tilde{\mu}_L}^2 + f_{21}^2$$

$$q_\mu = m_{\tilde{\mu}_R}^2 + f_{21}^2$$

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<sup>1</sup>This is the case if the angle is to lie in the first quadrant. As we shall see in detail in the next section, there can be nontrivial solutions for  $\theta_0$  in the second quadrant also.

$$t_\tau = (-v_1^2 + v_2^2 + v_\tau^2 - v_\mu^2)$$

$$t_\mu = (-v_1^2 + v_2^2 - v_\tau^2 + v_\mu^2)$$

$$\frac{1}{4}g'^2 c = \alpha'_c; \quad \frac{1}{2}g^2 v_1 v_2 = \alpha_{12}; \quad \frac{1}{2}g^2 v_1 v_\tau = \alpha_{1\tau}; \quad \frac{1}{2}g^2 v_2 v_\tau = \alpha_{2\tau}$$

$$\frac{1}{2}g^2 v_2 v_\mu = \alpha_{2\mu}; \quad \frac{1}{2}g^2 v_1 v_\mu = \alpha_{1\mu}; \quad \frac{1}{4}g^2 t_\tau = \alpha_{t\tau}; \quad \frac{1}{4}g^2 t_\mu = \alpha_{t\mu}$$

$$f_3 v_\tau = f_{3\tau}; \quad f_3 v_1 = f_{31}; \quad f_3 v_2 = f_{32}$$

$$f_2 v_\mu = f_{2\mu}; \quad f_2 v_1 = f_{21}; \quad f_2 v_2 = f_{22}$$

where both the left- and the right-chiral sleptons for each flavour have been included, the basis being  $(H_1, H_2, \tilde{\tau}_L, \tilde{\mu}_L, \tilde{\tau}_R, \tilde{\mu}_R)$ . Similarly, in the bases  $(Re(H_1), Re(H_2), Re(\tilde{\nu}_\tau), Re(\tilde{\nu}_\mu))$  and  $(Im(H_1), Im(H_2), Im(\tilde{\nu}_\tau), Im(\tilde{\nu}_\mu))$  respectively, the neutral scalar and the pseudoscalar mass matrices are given by

$$M_s^2 = \begin{pmatrix} m_1^2 + 2\lambda c + 4\lambda v_1^2 & -4\lambda v_1 v_2 + B\mu & 4\lambda v_1 v_\tau + \mu\epsilon_3 & 4\lambda v_1 v_\mu + \mu\epsilon_2 \\ -4\lambda v_1 v_2 + B\mu & m_2^2 - 2\lambda c + 4\lambda v_2^2 & -4\lambda v_2 v_\tau + B_3\epsilon_3 & -4\lambda v_2 v_\mu + B_2\epsilon_2 \\ 4\lambda v_1 v_\tau + \mu\epsilon_3 & -4\lambda v_2 v_\tau + B_3\epsilon_3 & m_{\tilde{\nu}_\tau}^2 + 2\lambda c + 4\lambda v_\tau^2 & \epsilon_2\epsilon_3 + 2\lambda v_\mu v_\tau \\ 4\lambda v_1 v_\mu + \mu\epsilon_2 & -4\lambda v_2 v_\mu + B_2\epsilon_2 & \epsilon_2\epsilon_3 + 2\lambda v_\mu v_\tau & m_{\tilde{\nu}_\mu}^2 + 2\lambda c + 4\lambda v_\mu^2 \end{pmatrix} \quad (17)$$

and

$$M_p^2 = \begin{pmatrix} m_1^2 + 2\lambda c & -B\mu & \mu\epsilon_3 & \mu\epsilon_2 \\ -B\mu & m_2^2 - 2\lambda c & -B_3\epsilon_3 & -B_2\epsilon_2 \\ \mu\epsilon_3 & -B_3\epsilon_3 & m_{\tilde{\nu}_\tau}^2 + 2\lambda c & \epsilon_2\epsilon_3 \\ \mu\epsilon_2 & -B_2\epsilon_2 & \epsilon_2\epsilon_3 & m_{\tilde{\nu}_\mu}^2 + 2\lambda c \end{pmatrix} \quad (18)$$

It may be remarked that the charged scalar and the neutral pseudoscalar mass matrices will each have a zero eigenvalue, corresponding to the Goldstone bosons. Also, these scalar matrices have to be used for a complete determination of the allowed space in  $B_2$  and  $B_3$ , since a number of conditions related to electroweak symmetry breaking need to be satisfied before values of these parameters, as extracted from Eqns. (8) and (9), pass off as valid ones. These conditions [18] include the requirement that the potential be bounded from below, the negativity of the vev's and non-negativity of the eigenvalues. They can be obtained by a straightforward generalisation of the corresponding conditions given in reference 18 where lepton number violation in only one family has been considered.

### 3 The constraints

As we have already mentioned, our first constraint is on the quantity  $v'$ . The allowed range of  $\Delta m_{\mu\tau}^2$ , combining the fully contained events, partially contained events and upward-going muons, is about  $1.5 - 6.0 \times 10^{-3} \text{ eV}^2$  at 90% confidence level [19]. For the lightest neutralino mass varying between 50 and 200 GeV, this corresponds to  $v' = 0.0001 - 0.0003 \text{ GeV}$  (100–300 keV) to a fair degree of approximation. Since  $v' = \sqrt{v_\mu'^2 + v_\tau'^2}$ , this automatically puts a constraint on  $v_\mu'$  and  $v_\tau'$ , and hence on the vev's in basis 1 for each value of the  $\epsilon$ -parameters.

Next, the value of the angle  $\theta$  should determine  $v_\mu'$  and  $v_\tau'$  completely, once  $v'$  is fixed. As we have pointed out in the previous section, for each value of  $\epsilon_2/\epsilon_3$ , one is restricted to particular values of  $\theta$  in order that the condition of large angle mixing, namely  $\sin^2 2\theta_0 > 0.8$ , is satisfied in terms of the ultimate neutrino mixing angle  $\theta_0$ . In Figs. 1 and 2 we outline these allowed regions in the parameter space of  $\epsilon_2/\epsilon_3$  vs.  $\theta$ , for  $\epsilon_2/\epsilon_3 \leq 1$  and  $\epsilon_2/\epsilon_3 \geq 1$  respectively. The allowed bands are found to be sensitive to the ratio rather than the actual values of the  $\epsilon$ 's. The interesting point to note here is that there are two values of  $\theta$  for each value of the ratio along the x-axis. This is due to the two solutions with angles in either of the first or the second quadrant, both cases yielding the same value for the oscillation probability. Physically, the second case corresponds to the superposition of the  $\nu_\mu$  and  $\nu_\tau$  states being performed with an extra phase rotation through an angle  $\pi$  for one of them. In other words, the two solutions represent situations with the two neutrino flavours having identical and opposite CP-properties respectively.

The angles  $\theta$  and  $\theta_0$  are practically equal if  $\frac{\epsilon_2}{\epsilon_3} \ll \frac{m_\mu}{m_\tau}$ . In this case the off-diagonal elements of the charged lepton mass matrix  $\mathcal{M}_l$  in basis 2 are negligibly small compared to the diagonal ones. However, Figs. 1 and 2 clearly demonstrate that such a large hierarchy between the two bilinear terms in the superpotential is by no means a necessity and that in fact it is possible to have them not only with the same order of magnitude but also with an inverted hierarchy as well.

Each point in the allowed regions shown in Figs. 1 and 2 corresponds to a theory for which the two bilinear soft terms  $B_2$  and  $B_3$  can be calculated. The ranges of values thus obtained have been plotted in Figs. 3 and 4, again with opposite hierarchies of the parameters  $\epsilon_2$  and  $\epsilon_3$ . The two bands in each figure arise from the two solutions of  $\theta$  for each ratio of the  $\epsilon$ 's and a particular value of  $\epsilon_3$  (Fig. 1) or  $\epsilon_2$  (Fig. 2). This underlines the fact that the two solutions for the angle in the two quadrants are indeed physically distinct, with all the different phenomenological implications of the two combinations of the  $\mathcal{L}$ -violating soft terms <sup>2</sup>.

The value of  $B_2$  in Fig. 3 is found to become very large when  $\epsilon_2$  is assigned a value much smaller than that of  $\epsilon_3$  (and similarly for  $B_3$  in Fig. 4 with a very small  $\epsilon_3$ ). This is analogous to the  $\mu$ -B problem of the MSSM. However, Figs. 1 and 2 already tell us that such a large hierarchy of the

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<sup>2</sup>In fact, there could be two more sets of solutions in the two remaining quadrants. However, these solutions are not distinct from the previous ones, as they can be mapped back to the cases of same and opposite parity for the two neutrinos. Explicit computations also lead to the two already obtained combinations of  $B_2$  and  $B_3$ .

two  $\epsilon$ 's is *not* a necessary condition for large angle neutrino mixing. The very expressions for the  $B$ -parameters show that except in cases where very small  $\epsilon_{2(3)}$  jacks them up to large values, they are also controlled by the scale of the slepton mass. We also see from Fig. 3 that  $B_3$  cannot attain values much larger than the electroweak scale so long as  $\epsilon_2 \leq \epsilon_3$ . The same feature is observed for  $B_2$  in Fig. 4.

However, it must be noted at this point that the sets of values for  $B_2$  and  $B_3$  as seen in Fig. 3 or 4 result from the choices of two parameters, *viz.*,  $\epsilon_3$  (Fig.3) or  $\epsilon_2$  (Fig.4) and their appropriate ratios, the choices being rather specific and spanned over identical regions in these two figures. This is the reason why one of  $B_2$  and  $B_3$  ranges over several orders of magnitude while the other is highly constrained in a particular figure. Here, it should be made clear that the magnitudes of  $B_2$  and  $B_3$  are mainly determined by those of  $\epsilon_2$  and  $\epsilon_3$  respectively for very small values ( $< 10^{-5}$ ) of the latter two. As, neither the smallness of the individual values of  $\epsilon$ 's nor that of their ratios is anyway restricted, we can always tune these two parameters so that both of  $B_2$  (Eqn. (8)) and  $B_3$  (Eqn. (9)) are simultaneously very large (compared to the electroweak scale) leading to additional regions in the allowed  $B_2 - B_3$  space which are absent in Figs. 3 and 4.

Summing all these up, the parameter space of the soft terms (modulo their additional dependence on the MSSM parameters) is seen to be sufficiently free from any requirement of fine-tuning or large hierarchy, and large angle neutrino mixing as well as the expected neutrino mass hierarchy in the second and third families is reproduced over a rather wide range in the space of R-parity violating parameters. Also,  $B_2$  and  $B_3$  are not compelled to show any hierarchical behaviours with respect to the R-conserving soft term  $B$ , and any approach connecting them at a high scale is consistent with the range of values allowed here.

The discussion becomes more transparent if we break up the results shown in Figs. 3 and 4 into 3 broad regions, *viz.*, (i) very small  $\epsilon_2(\epsilon_3)$  ( $\sim 10^{-8}$ ), (ii) somewhat intermediate values of  $\epsilon_2(\epsilon_3)$  ( $\sim 10^{-5}$ ) and (iii) rather large values of  $\epsilon_2(\epsilon_3)$  ( $\sim 10^{-2}$ ) in reference to Eqn. (8) (Eqn. (9)). The numbers in the parenthesis correspond to the left extreme graphs of Fig. 3 (Fig. 4). In case (i)  $\epsilon_2(\epsilon_3)$  dependent terms lead and its presence in the denominators with a very small value leads to a large value for  $B_2(B_3)$ . When  $\epsilon_2(\epsilon_3)$  increases as for case (ii) the contributions from these terms gradually become comparable to that coming from the 4th term which depends only upon MSSM parameters, *viz.*,  $\mu$  and  $\tan\beta$ . This restricts  $B_2(B_3)$  to intermediate values. The relative sign between the 4th term and other terms collectively is instrumental in fixing the sign of  $B_2(B_3)$  in this region and is clearly visible as we proceed along the second row of graphs in Fig. 3 (Fig. 4). In case (iii) the 4-th term controls  $B_2(B_3)$ . Naturally, in this case, the order of  $B_2(B_3)$  is set by  $\mu$  and  $\tan\beta$  and is restricted to be around 100 GeV for choices of these two parameters that are compatible with existing collider data.

In the results presented above,  $\mu$  and  $\mu'$  have been taken to be of the same sign, with both  $\epsilon_2$  and  $\epsilon_3$  having signs opposite to it. On reversing this relative sign (and also that between the two  $\epsilon$ -terms), it is seen that the orders of magnitude of  $B_2$  and  $B_3$  do not change; however, the signs of

one or both of them are liable to get reversed.

On reversing the sign of  $\mu$ , both  $B_2$  and  $B_3$  may change sign if the bulk contributions to them come from the 4-th terms in Eqns. (8) and (9). Otherwise, the relative sign between  $B_2$  and  $B_3$  and also their magnitudes (to a good approximation) are preserved under such reversal. Changing the sign of  $\mu$  should be accompanied by a change in sign of  $B$  (of MSSM) to retain a relative sign between themselves which is a must to achieve electroweak symmetry breaking at the proper scale. On the other hand,  $B_2$  and  $B_3$  pick up a relative sign on allowing for the same between  $\epsilon_2$  and  $\epsilon_3$  only if the latter two contribute heavily to  $B_2$  and  $B_3$  respectively.

We restrict ourselves by presenting only two sets of figures (Figs. 3 and 4) which jointly cover rather wide intervals of  $\epsilon_2$  and  $\epsilon_3$ . Results of any combinatorics of relative signs between  $\mu$  and the  $\epsilon$ 's can be directly estimated to a fair degree of accuracy from these two sets of figures. These show that the magnitudes of  $B_2$  and  $B_3$  can range over several orders of magnitude with any relative sign between them.

It should be re-iterated that variations of  $B_2$  ( $B_3$ ) with the MSSM parameters  $\mu$  and  $\tan\beta$  are likely to be most pronounced when major contributions to it comes from the 4th term in Eqn. (8) (Eqn. (9)). This should be contrasted with the results presented in Figs. 1 and 2 where the allowed band is quite insensitive to the MSSM parameters.  $B_2$  and  $B_3$  exhibits such a sensitivity when  $\epsilon_2$  ( $\epsilon_3$ ) is  $\simeq 10^{-3}$  or more, so that the other terms in Eqn. (8) (Eqn. (9)) have comparable orders of magnitude ( $\simeq$  electroweak scale) leading to  $B_2$  ( $B_3$ ) lying around 100 GeV. Here, variation of  $\tan\beta$  over a range 5 to 50 can change  $B_2$  ( $B_3$ ) at most by 1 order while variation with  $\mu$  over a range  $-500$  to  $+500$  does not change things much, although a flip in the relative sign between  $B_2$  and  $B_3$  may be a possible outcome of the variation of  $\mu$ .

In the above, we have not discussed the mass-splitting required between the electron and the muon neutrinos for explaining the solar neutrino problem. As we have already mentioned, the trilinear interactions in Eqn. (2) give rise to loop-induced masses that can account for this splitting rather well. The nature of the splitting will depend on whether one wants to provide an explanation in terms of matter-enhanced or vacuum oscillations, and the only consequent constraints will come on the respective  $\lambda$ -and  $\lambda'$ -couplings in basis  $\mathcal{Z}$ .

There are, however, additional issues that must be addressed here. In general, the very structure of the charged scalar, neutral scalar and pseudoscalar mass terms, obtainable from Eqn. (3), admits of mixing between the smuon and stau flavour states, as also between the corresponding sneutrino flavours in both the neutral scalar and pseudoscalar mass matrices. This in general causes a mismatch between these scalar mass matrices and the charged lepton mass matrix, standing in the way of their simultaneous diagonalisation. This in general can lead to an enhancement of FCNC, contributing to processes like  $\tau \longrightarrow \mu\gamma$ .

FCNC suppression can in general be ensured by one of three mechanisms— degeneracy, alignment and decoupling [20]. Decoupling in this case requires the slepton and sneutrino masses to be very high so that contributions to the relevant processes from loop diagrams can be suppressed. With

these masses on the order of 100 GeV, one has to ensure either alignment of the scalar mass matrices with the one for charged leptons, or a close degeneracy of the scalar eigenvalues. Both conditions are seen to be satisfied in this case if  $(M_{\tilde{L}_2\tilde{L}_3}^2)_i \ll (M_{\tilde{L}_2\tilde{L}_2}^2)_i$  for  $i = s, p \text{ or } c$  (i.e. in the neutral scalar, pseudoscalar or charged scalar mass squared matrices).

Figures 5 and 6 contain plots of the ratio of the  $\mu_L\tau_L$  and  $\mu_L\mu_L$  elements for each of the three matrices, against the ratio of  $\epsilon_2$  and  $\epsilon_3$  (again for both  $\epsilon_2 \leq \epsilon_3$  and  $\epsilon_2 \geq \epsilon_3$ ). A pattern is revealed in each case on imposing the requirement on the off-diagonal elements to be small compared to the diagonal ones.

First consider the case  $\epsilon_2 \leq \epsilon_3$ . For small values of this ratio, the smallness of all the diagonal terms (and therefore the suppression of FCNC) is assured, irrespective of the value of  $\epsilon_3$ . In such cases it is enough to consider the scalar mass matrices in basis  $\mathcal{L}$  itself, since the condition  $\epsilon_2 \ll \epsilon_3$  makes the charged lepton mass matrix practically diagonal in this basis. The off-diagonal terms in this basis are controlled by the quantity  $v'^2$  which is constrained from neutrino masses to be much smaller than  $m_l^2$  and  $\mu^2$  which dominate the values of the diagonal terms. However, as one increases  $\epsilon_2/\epsilon_3$ , the charged lepton mass matrix ceases to be diagonal in basis  $\mathcal{L}$ , and the scalar mass matrices need to be evaluated *after* diagonalising it. In such cases the off-diagonal elements are also influenced by the actual value of  $\epsilon_3^2$ . As can be seen from Fig. 5, for  $\epsilon_3$  approaching 100 GeV, the off-diagonal term starts becoming comparable to the diagonal ones as the ratio  $\epsilon_2/\epsilon_3$  is close to unity. Exactly the same thing can be said of  $\epsilon_2$  in Fig. 6 where predictions for an inverted ratio between the two  $\epsilon$ 's are shown.

A rather interesting conclusion follows from the above discussion. While a compatibility with the SK results is generally observed over a large area in the space of R-parity violating parameters, the need to suppress FCNC effects introduces at least one type of hierarchy. This is apparent from the fact that one of the two bilinear parameters  $\epsilon_2$  and  $\epsilon_3$  can be allowed to be as large as in the electroweak scale if their mutual ratio has a hierarchy of 2 to 3 orders of magnitude. On the other hand, allowing the two of them to be of the same order is consistent with the suppression of FCNC only if the scale of their magnitude is small compared with the corresponding R-conserving parameter i.e.  $\epsilon_{2(3)} \ll \mu$ . Thus, either a hierarchy in the two R-violating parameters in the superpotential or a smallness of the R-violating ones compared to R-conserving ones is a necessary consequence of this scenario.

## 4 Conclusion

We have investigated a SUSY theory where the bilinear R-parity violating terms are responsible for tree-level neutrino mass and large angle mixing, leading to a pattern that explains the observed deficiency of atmospheric muonic neutrinos. The scenario also admits of trilinear R-parity violating terms that cause the mass splitting necessary to explain the solar neutrino puzzle.

A detailed study of the parameter space of the theory shows that, as far as constraints arising

from the mass-splitting and large angle mixing are concerned, neither a hierarchy of R-violating parameters nor a fine-tuning among them is necessary, and a large area of the parameter space of the R-breaking soft terms is ‘naturally’ allowed. However, the suppression of FCNC can require either the R-violating parameters in the superpotential to be small compared to the  $\mu$ -parameter, or such parameters themselves to have a mutual hierarchy of values.

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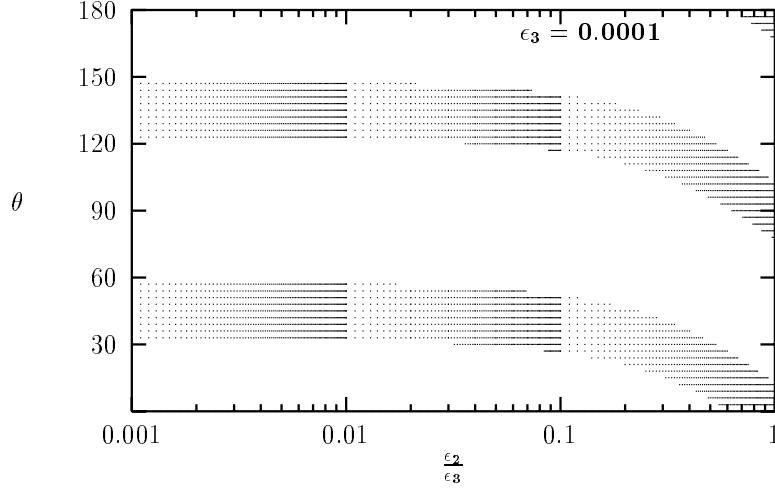


Figure 1: Allowed bands of neutrino mixing angle ( $\theta$  in degrees) corresponding to maximal mixing between 2nd and 3rd generation of neutrinos are plotted against the ratio of epsilons ( $\frac{\epsilon_2}{\epsilon_3}$ ). The MSSM parameters are  $\mu = -500$ ,  $\tan \beta = 10$  and  $B = 100$ . The pattern remains almost the same for different  $\epsilon_3$ 's. The different shadings arise due to varying number of sample points in different regions.

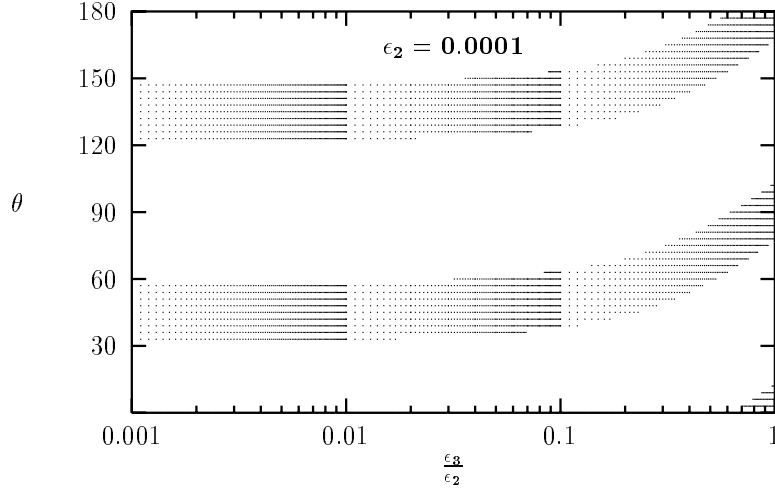


Figure 2: Allowed bands of neutrino mixing angle ( $\theta$  in degrees) corresponding to maximal mixing between 2nd and 3rd generation of neutrinos is plotted against the ratio of epsilons ( $\frac{\epsilon_3}{\epsilon_2}$ ). The MSSM parameters are same as in Fig. 1. The pattern remains almost the same for different  $\epsilon_2$ 's. The different shadings arise due to varying number of sample points in different regions.

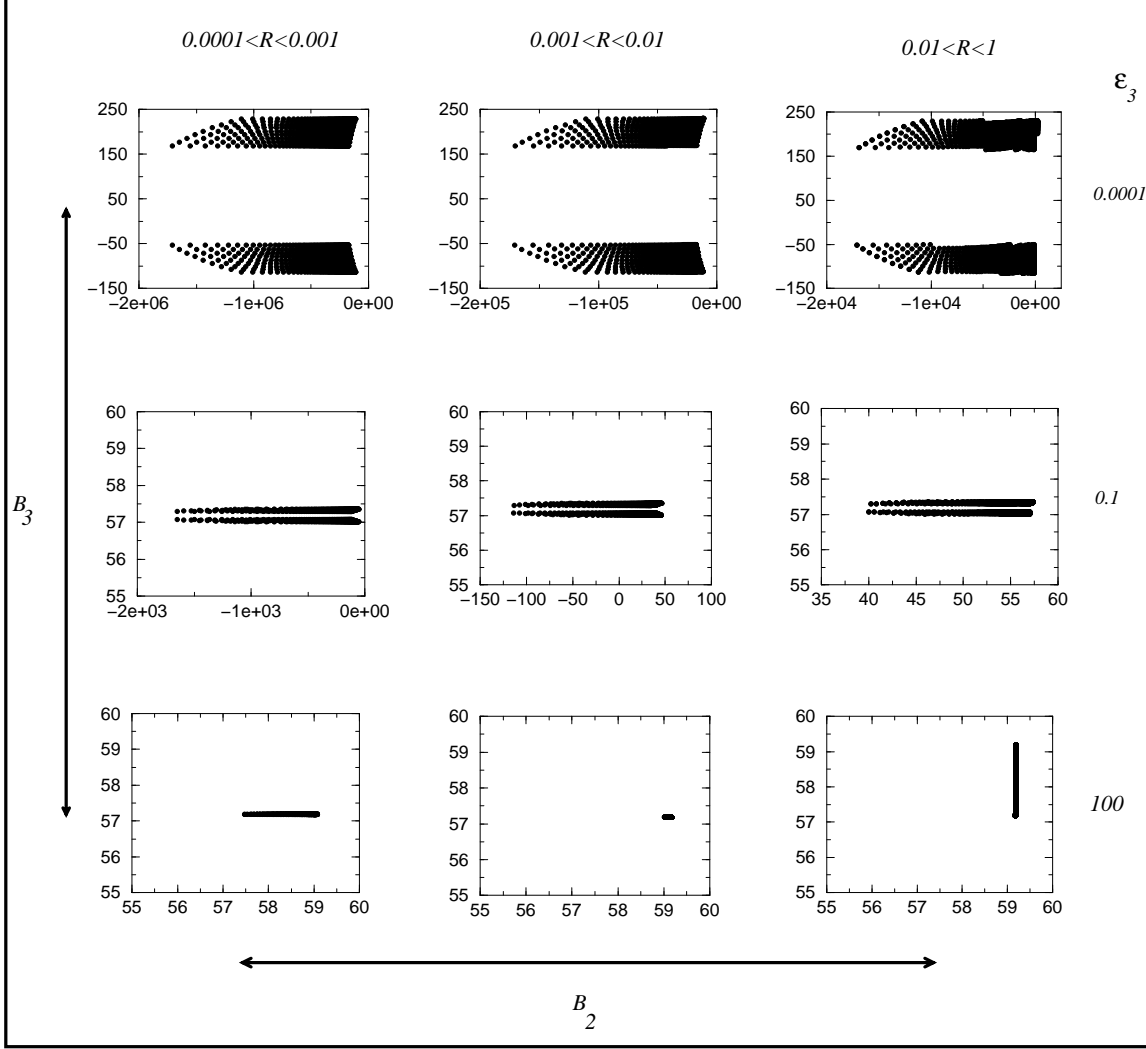


Figure 3: Allowed regions of  $B_2$  -  $B_3$  space (in GeV).  $R = \frac{\epsilon_2}{\epsilon_3}$  varies along the columns as indicated in the top margin.  $\epsilon_3$  varies along the rows as shown in the right margin. The MSSM parameters are  $\mu = -500$ ,  $\tan \beta = 10$  and the common slepton mass at the electroweak scale is 200 GeV.

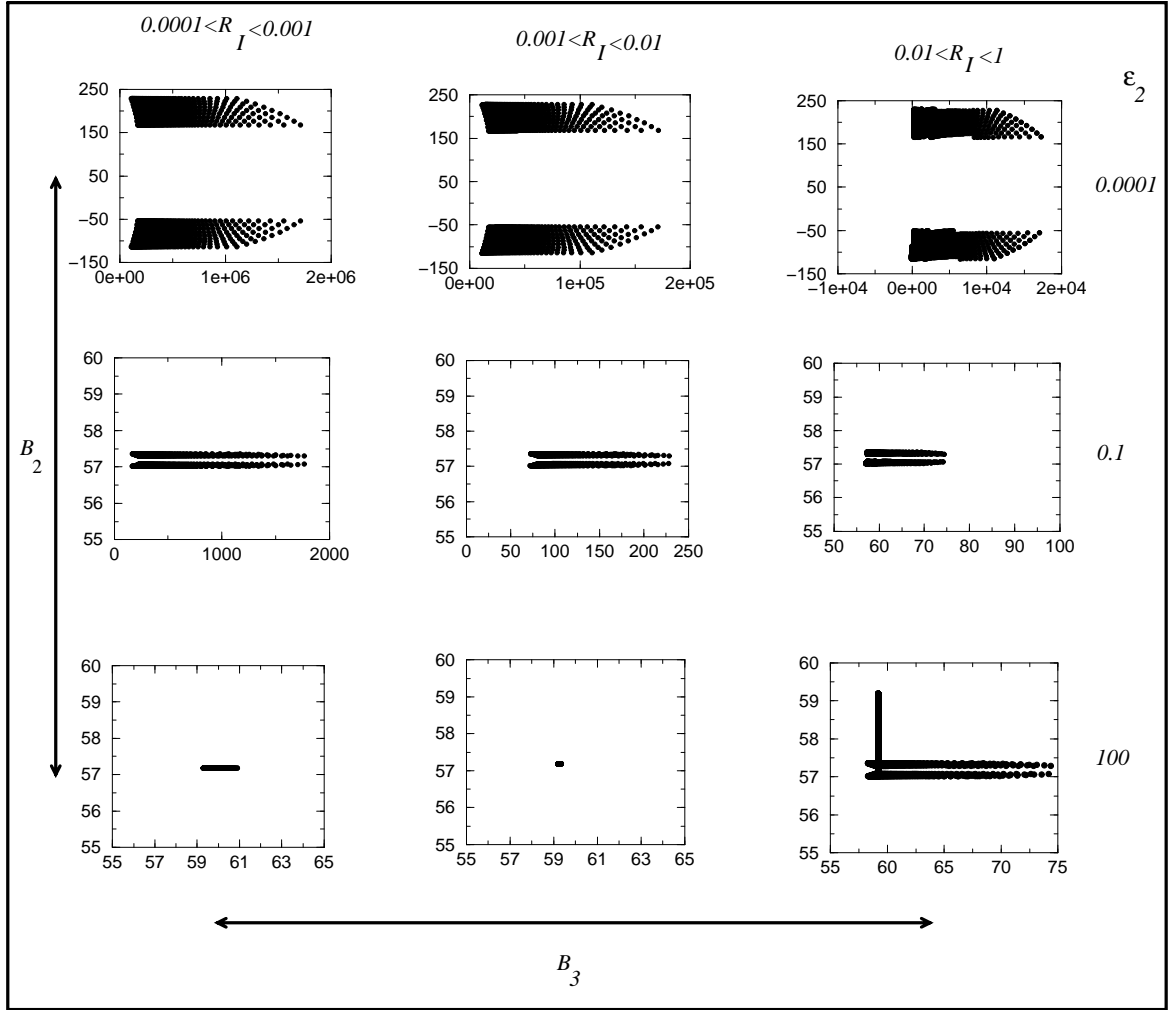


Figure 4: Allowed regions of  $B_2 - B_3$  space (in GeV).  $R_I = \frac{\epsilon_3}{\epsilon_2}$  varies along the columns as indicated in the top margin.  $\epsilon_2$  varies along the rows as shown in the right margin. The MSSM parameters are same as in Fig. 3.

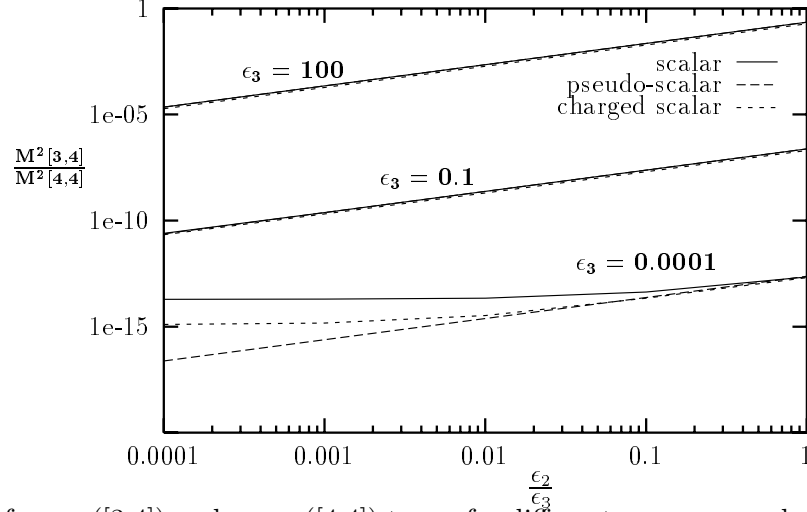


Figure 5: Ratio of  $\mu_L \tau_L$  ([3,4]) and  $\mu_L \mu_L$  ([4,4]) terms for different mass-squared matrices (in a basis where the lepton mass matrix is diagonal) is plotted against the ratio  $(\frac{\epsilon_2}{\epsilon_3})$ . The MSSM parameters are  $\mu = -500$ ,  $\tan \beta = 10$ ,  $B = 100$  and common slepton mass at the electroweak scale is 200 GeV.

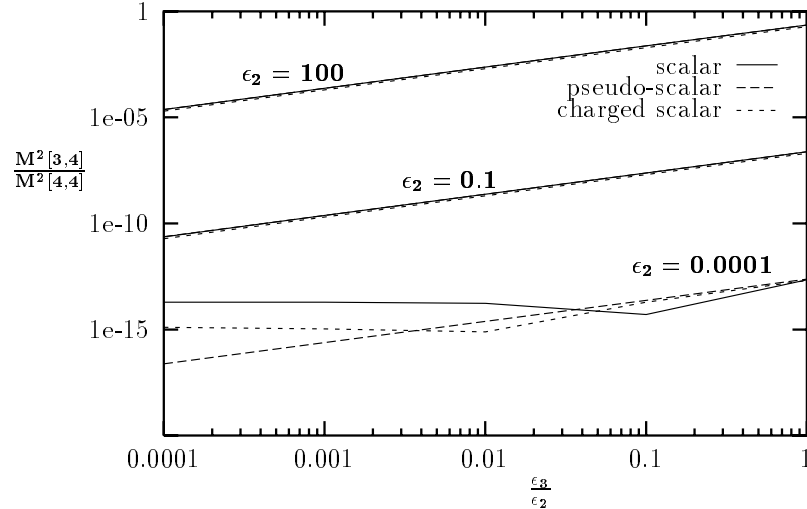


Figure 6: Ratio of  $\mu_L \tau_L$  ([3,4]) and  $\mu_L \mu_L$  ([4,4]) terms for different mass-squared matrices (in a basis where the lepton mass matrix is diagonal) is plotted against the ratio  $(\frac{\epsilon_3}{\epsilon_2})$ . The MSSM parameters are same as in Fig. 5.